GLM and GAMs Workshop

By Aaron Greenville

- Stats model
- Distributions
- GLM and GLMM
  - Over dispersion
  - Temporal autocorrelation
- GAM and GAMM
  - Random variables
  - Spatial autocorrelation

Stats model

DETERMINISTIC

STOCHASTIC

$$mass_i = \alpha + \beta \times Sex_i + \varepsilon_i$$

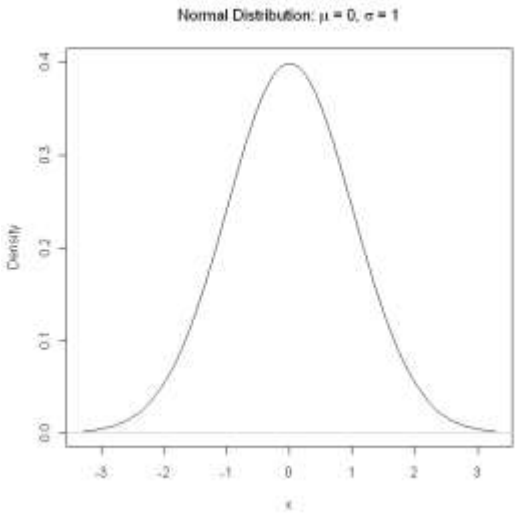
Constants

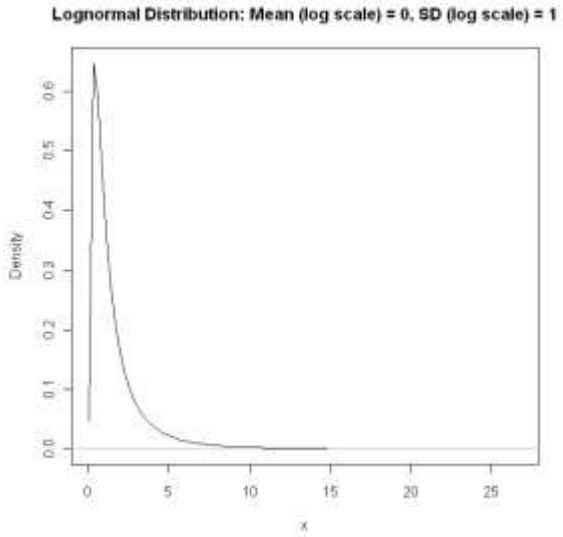
We are used to ε_i following a normal distribution

Remember linear equation...

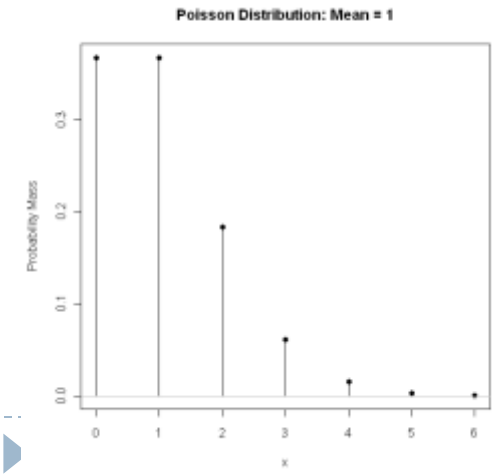
Beyond the normal distribution

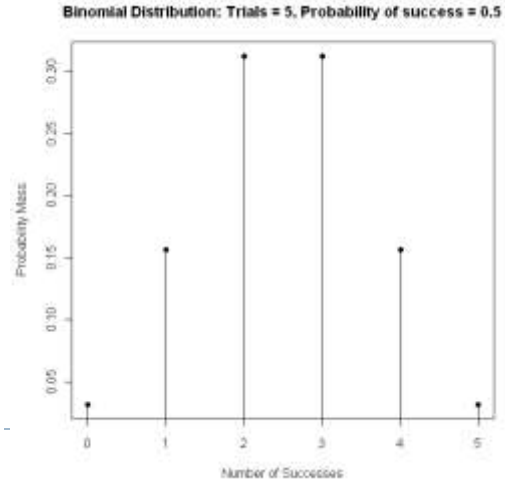
Continuous distributions

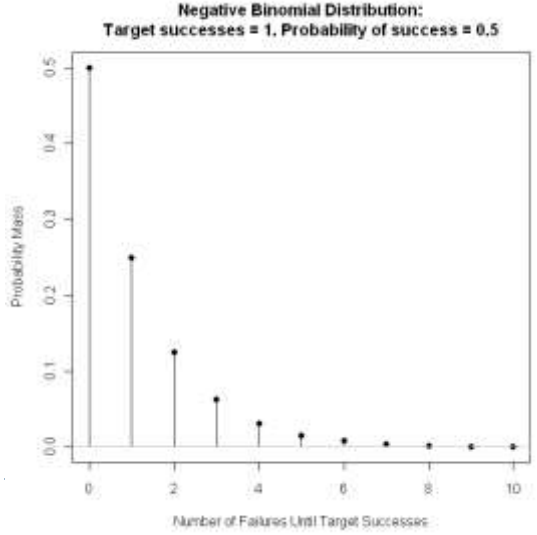




Discrete distributions







Generalized linear models (GLM)

- We choose the distribution the error (stochastic part) follows. Hence Generalized.
- Very powerful as they are flexible
  - Binomial regression - the probability of a success is related to explanatory variables: the corresponding concept in ordinary regression is to relate the mean value of the unobserved response to explanatory variables.
    - Logistic regression - is used for prediction of the probability of occurrence of an event by fitting data to a logistic curve. Special case of binomial regression
  - Poisson or negative binomial models
  - Zero-inflated models

GLM cont.

- Quasi-distributions
- Can have random variables, nested designs etc
- Can use traditional hypothesis testing
- Or model selection techniques (AICc's etc)
- Can use Bayesian methods

GLM cont.

- Link function
  - Specify the relationship of the response variable (y) and deterministic part (predictor variables)

- So GLM has 3 parts
  - Data follows some dist e.g mass follows Poisson, mean = variance.
  - Link between mean of y (mass) and predictor variable(s). E.g. Log for poisson
  - Deterministic part: $\log(\text{mean mass}_i) = \alpha + \beta \times \text{Sex}_i$

- Deviance = (null deviance – residual deviance)/null deviance

Poisson GLM example: Frog roadkill

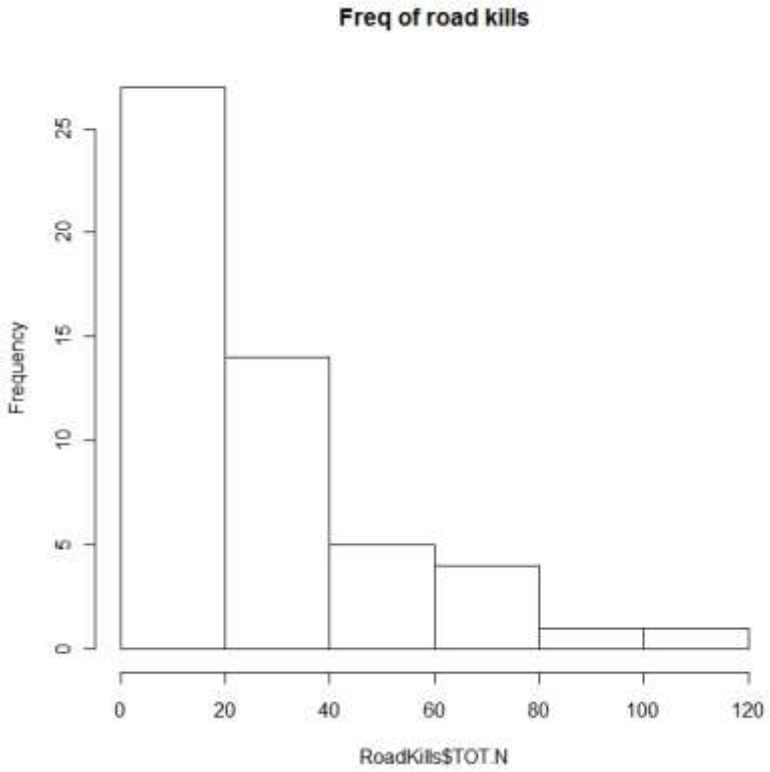
Exercise 5:

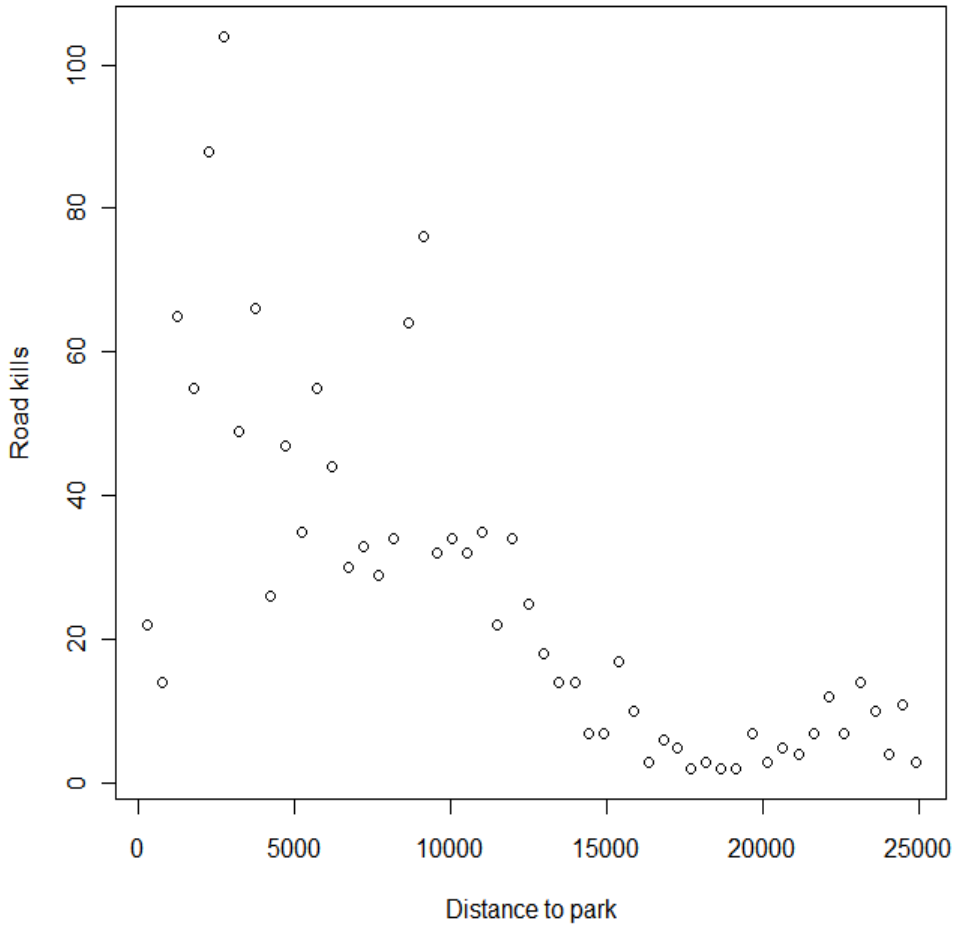
1. No. of frogs killed follows Poisson dist

2. log link function needed

3. log(mean frogsKilled)= $\alpha + \beta$ x Dist.Park+ ε_i

GLM cont.: Frog road kill





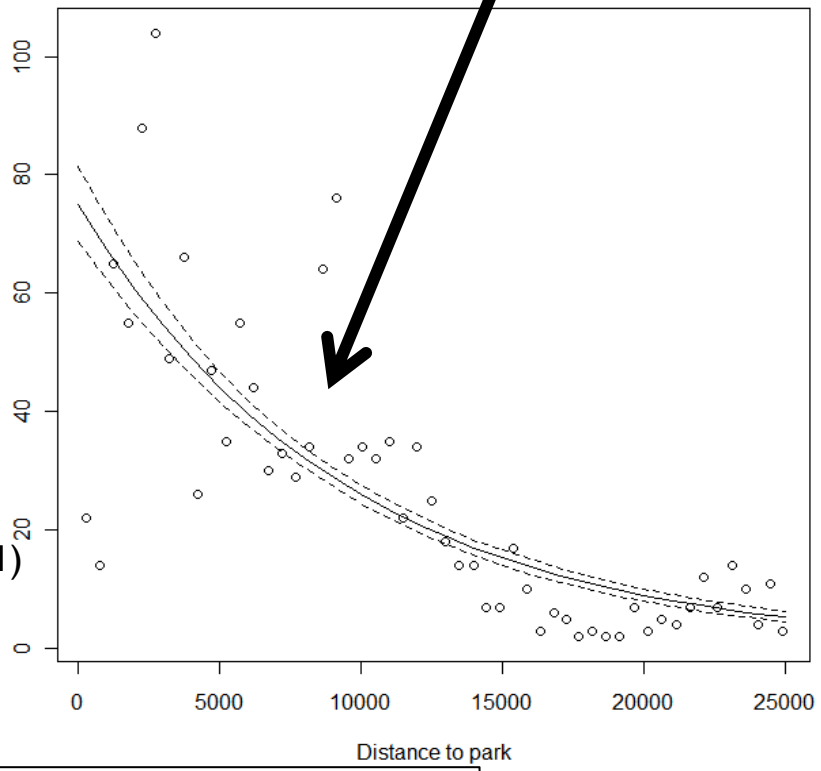
Poisson GLM example: Frog roadkill

```
glm(formula = TOT.N ~ D.PARK, family = poisson, data = RK)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-8.1100  -1.6950  -0.4708   1.4206   7.3337

Coefficients:
             Estimate   Std. Error   z value   Pr(>|z|)
(Intercept)  4.316e+00  4.322e-02    99.87     <2e-16 ***
D.PARK       -1.059e-04 4.387e-06    -24.13    <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 1071.4  on 51  degrees of freedom
Residual deviance:  390.9  on 50  degrees of freedom
AIC: 634.29
~64% deviance explained
```

α

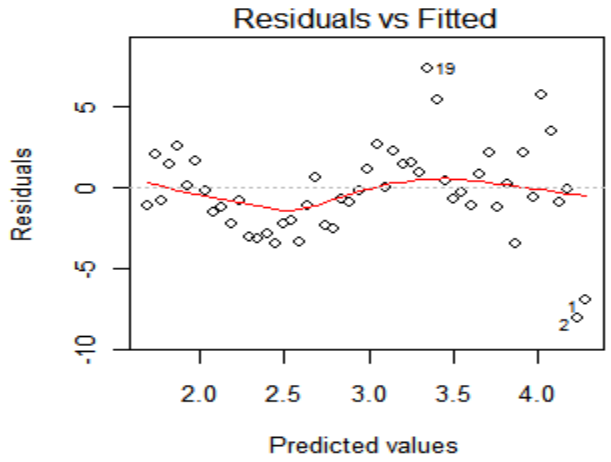
β

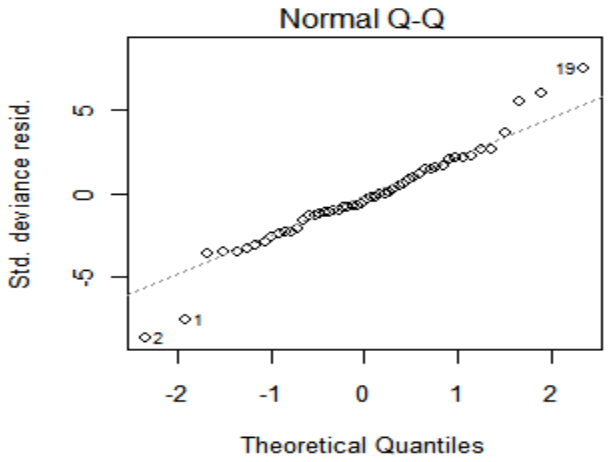
Not linear because of the log link function

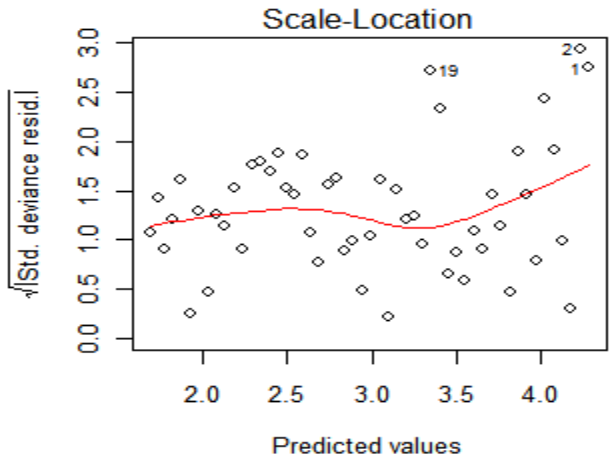


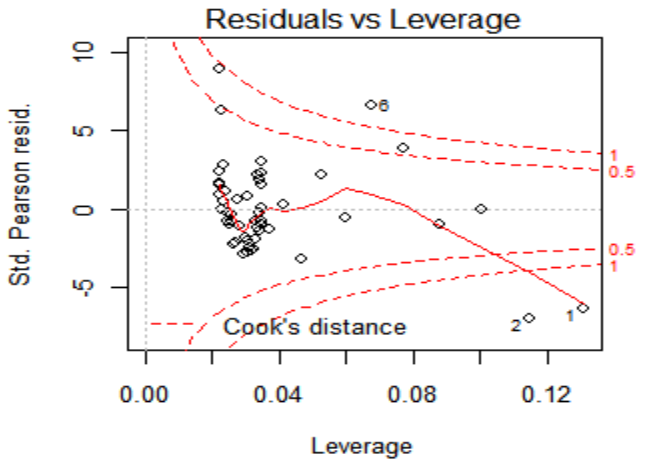
Looks like over-dispersion here

GLM cont.: model checking









Quasi-poisson GLM

glm(formula = TOT.N ~ D.PARK, family = quasipoisson, data = RK)

Deviance Residuals:

Min	1Q	Median	3Q	Max
-8.1100	-1.6950	-0.4708	1.4206	7.3337

Coefficients:

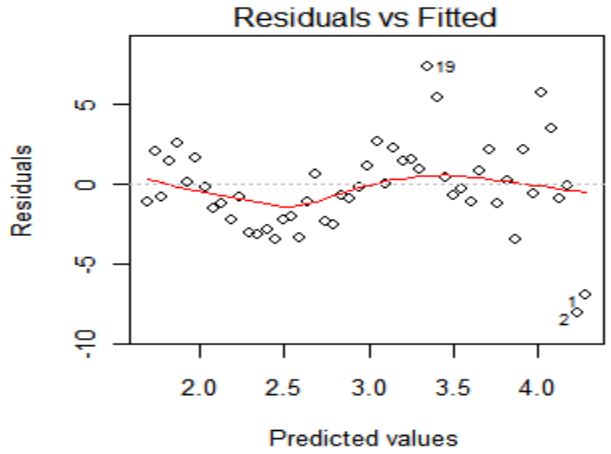
	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.316e+00	1.194e-01	36.156	< 2e-16 ***
D.PARK	-1.058e-04	1.212e-05	-8.735	1.24e-11 ***

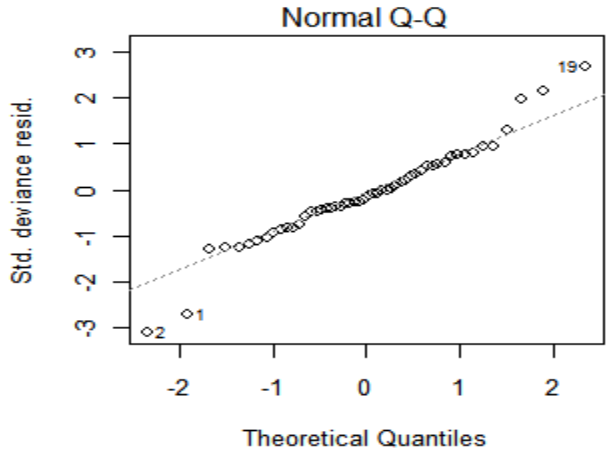
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

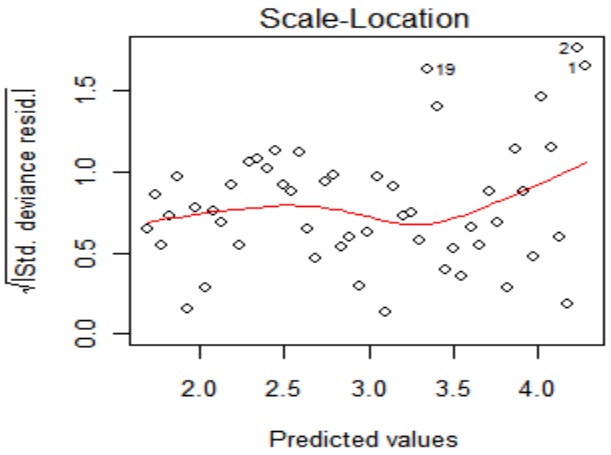
(**Dispersion parameter for quasipoisson family taken to be 7.630148**)

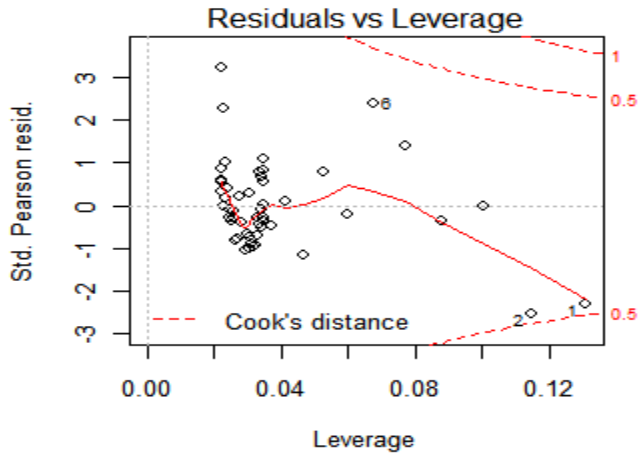
Null deviance: 1071.4 on 51 degrees of freedom
Residual deviance: 390.9 on 50 degrees of freedom
AIC: NA

GLM cont.: model checking









Neg bin GLM: Frog road kill

glm.nb(formula = TOT.N ~ D.PARK, data = RK, link = "log", init.theta = 3.681040094)

Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.4160	-0.8289	-0.2116	0.4800	2.1346

Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	4.411e+00	1.548e-01	28.50	<2e-16 ***
D.PARK	-1.161e-04	1.137e-05	-10.21	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(3.681) family taken to be 1)

Null deviance: 155.445 on 51 degrees of freedom

Residual deviance: 54.742 on 50 degrees of freedom

AIC: 393.09

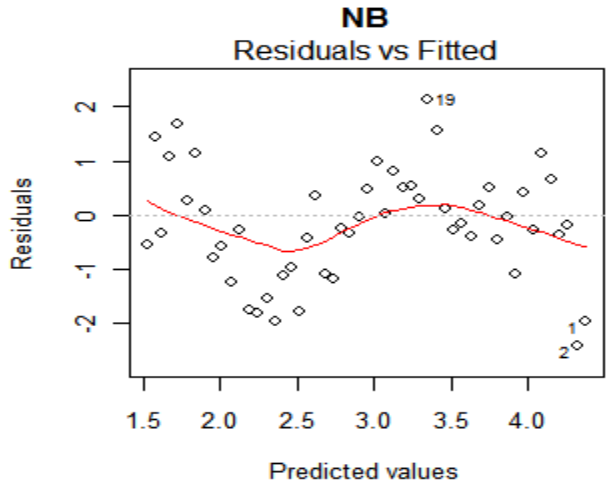
Number of Fisher Scoring iterations: 1

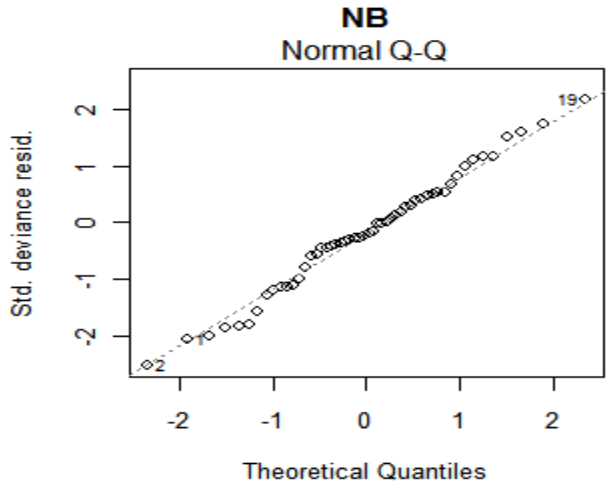
Theta: 3.681

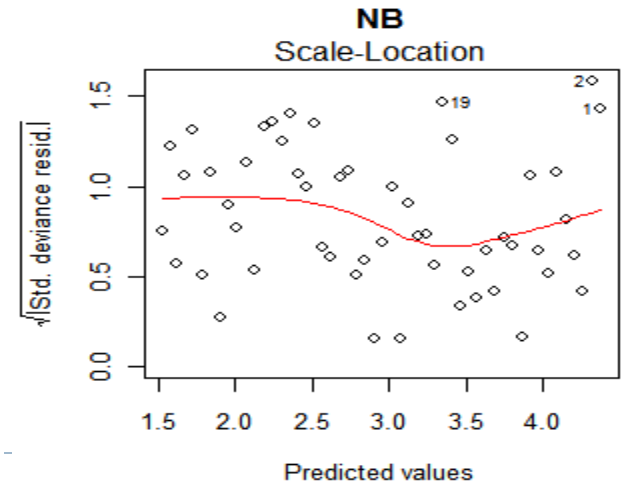
Std. Err.: 0.891

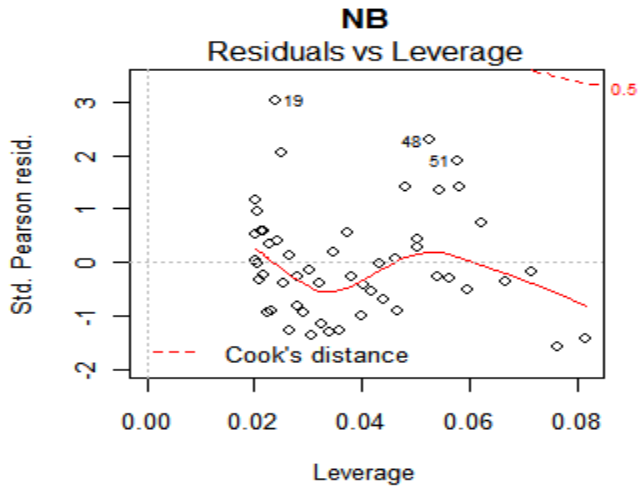
~65% deviance explained

GLM cont.: model checking







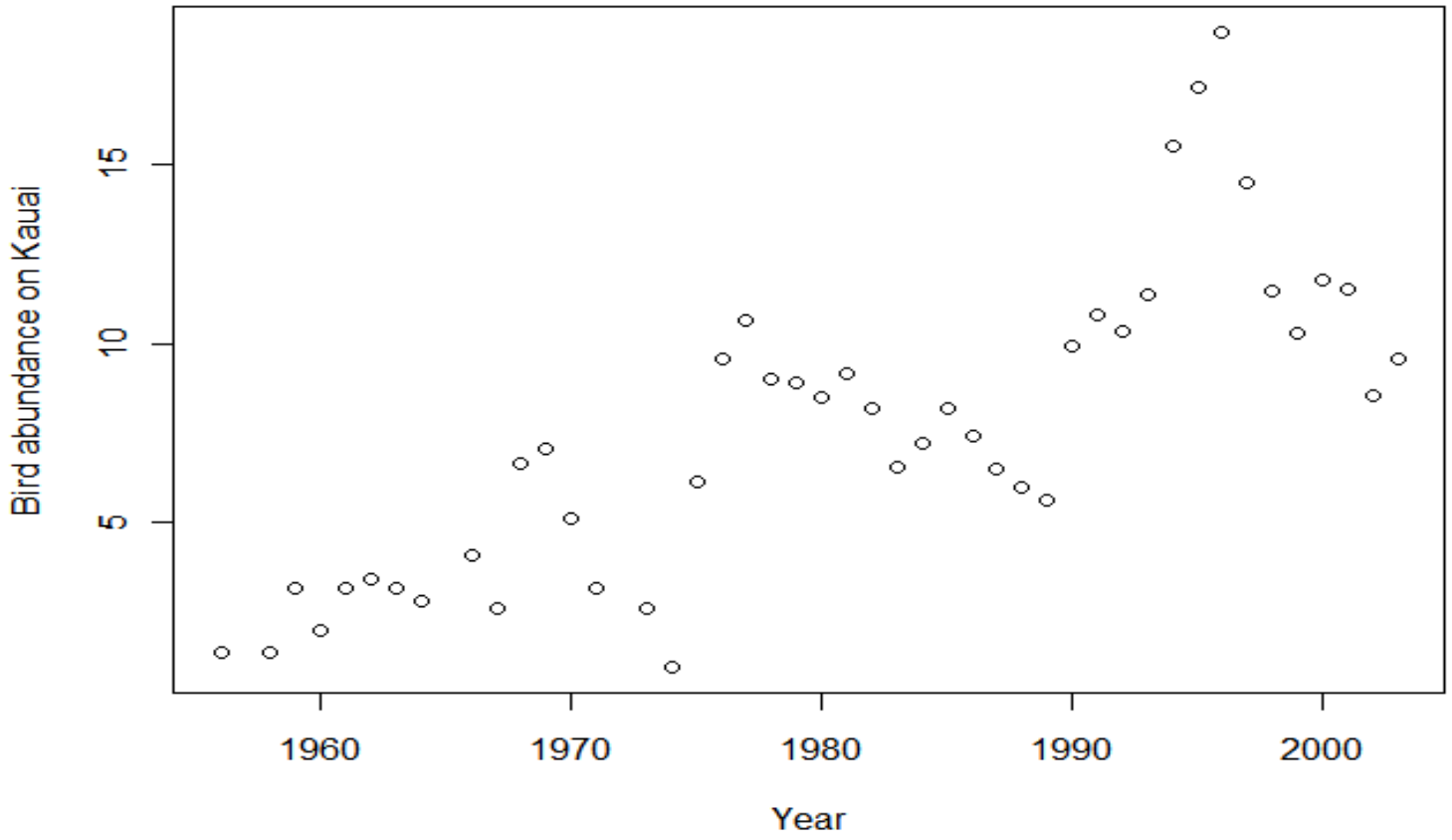


GLMM with temporal confounding

Exercise 6:

- Hawaii birds abundance over time
- Normal dist with identity link function
- $\textit{Mean birds} = \alpha + \beta \times \textit{Year} + \beta_2 \textit{Rainfall} + \varepsilon_i$

GLMM: Bird e.g cont.



GLMM: Birds e.g. cont.

Generalized least squares fit by REML

Model: Birds ~ Rainfall + Year

Data: Hawaii

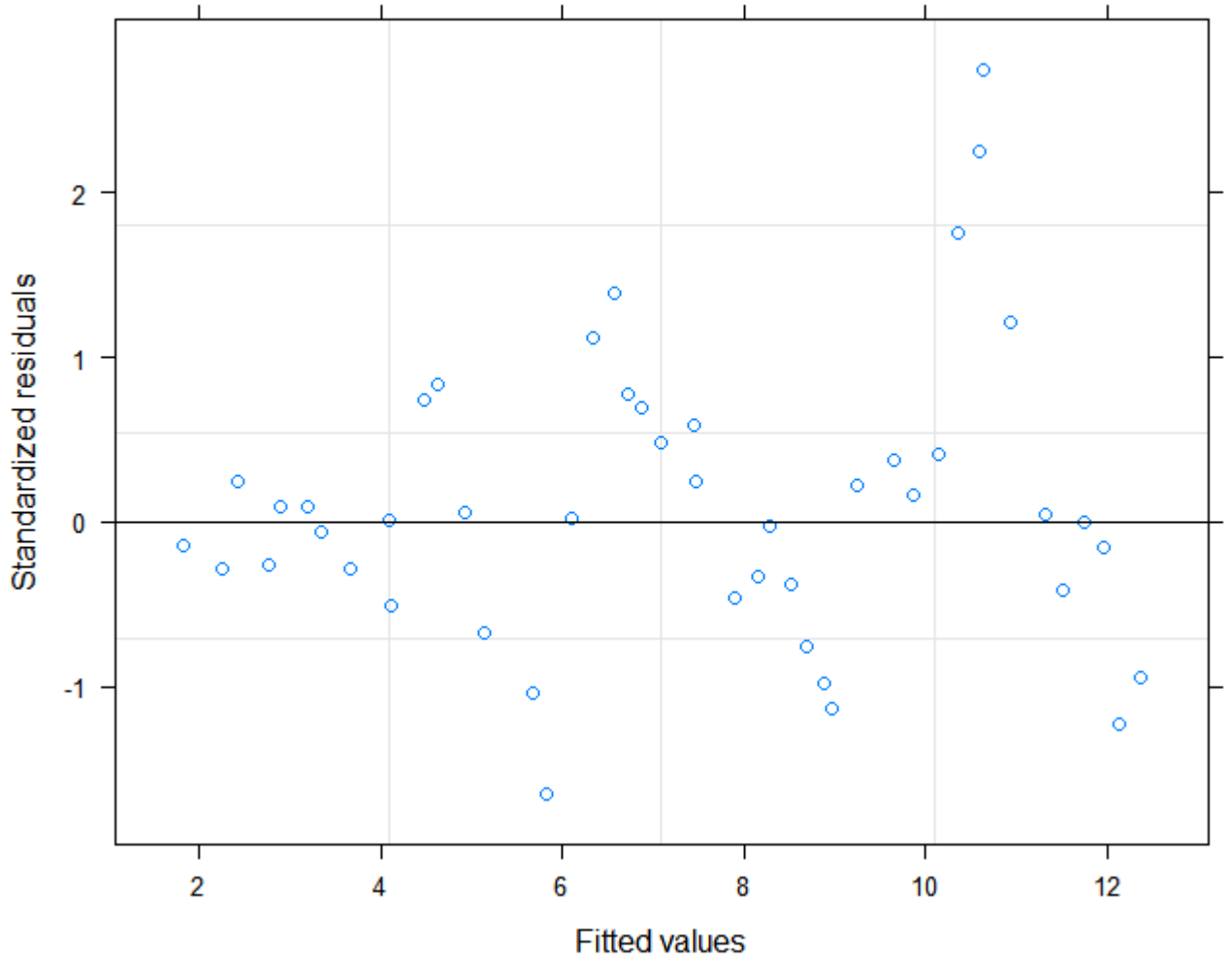
AIC	BIC	logLik
228.4798	235.4305	-110.2399

Coefficients:

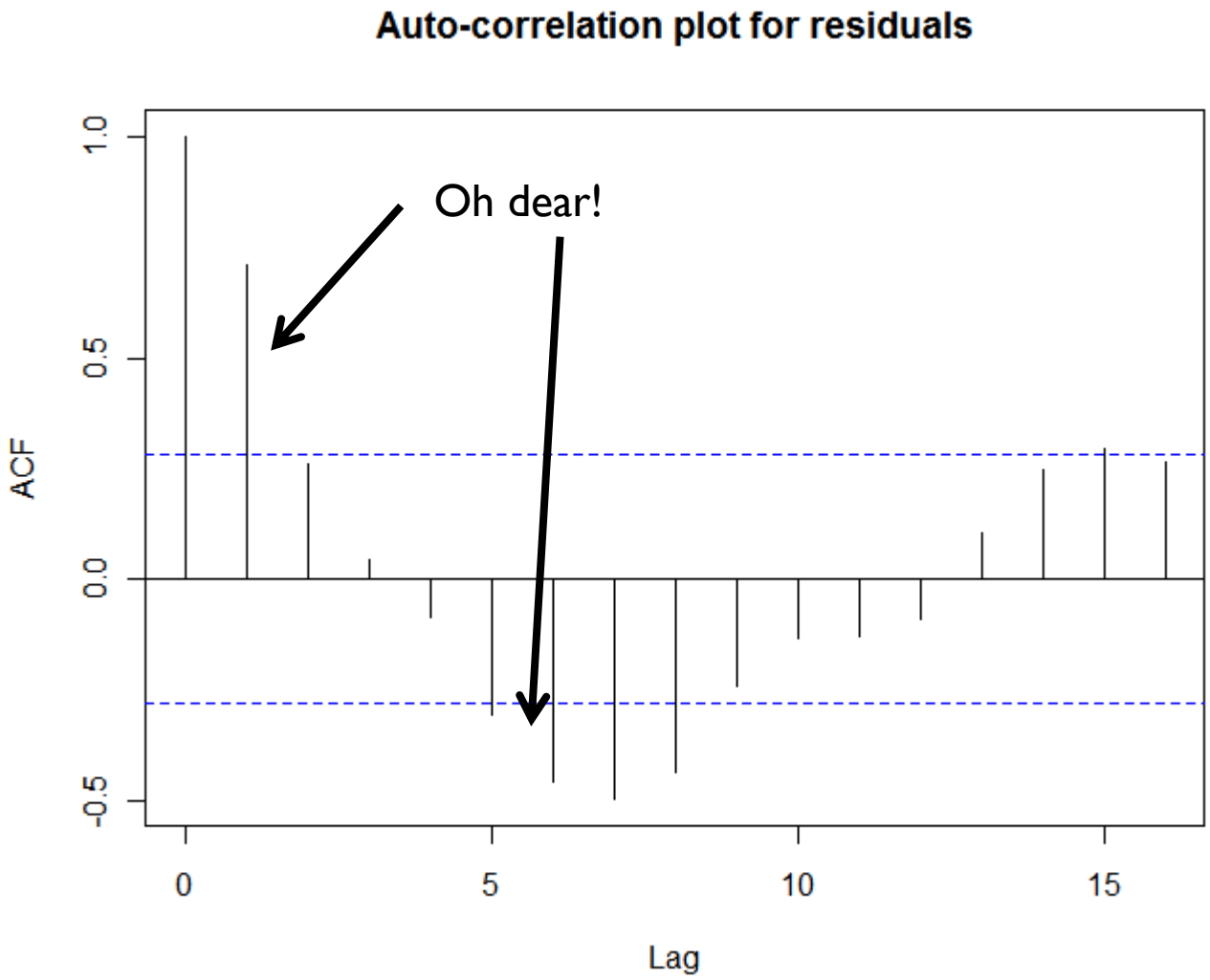
	Value	Std.Error	t-value	p-value
(Intercept)	-477.66	56.41907	-8.466346	0.0000
Rainfall	0.0009	0.04989	0.017245	0.9863
Year	0.2450	0.02847	8.604858	0.0000

GLMM cont.

Note pattern



Looking for temporal autocorrelation



GLMM cont.

- Need to take into account temporal autocorrelation/confounding

- Lots of variance structures you can use.
  - corAR1: Says data 1 yr apart is more correlated than 2 yrs apart, 3 yrs apart etc. So after x number of years there will be no correlation.
  - corARMA: autoregressive moving average process, with arbitrary orders for the autoregressive and moving average components.
  - corCAR1: continuous autoregressive process (AR(1) process for a continuous time covariate).
  - corCompSymm: compound symmetry structure corresponding to a constant correlation.

GLMM cont.

Generalized least squares fit by REML
Model: Birds ~ Rainfall + Year
Data: Hawaii

AIC	BIC	logLik
199.1394	207.8277	-94.5697

AIC lower

Correlation Structure: ARMA(1,0)
Formula: ~Year
Parameter estimate(s):

Phi1
0.7734303

Residuals separated by 1 yr are correlated at 0.77, 2 yrs 0.77^2 etc

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-436.4326	138.74948	-3.145472	0.0030
Rainfall	-0.0098	0.03268	-0.300964	0.7649
Year	0.2241	0.07009	3.197828	**0.0026**

p-value not as sign.

Generalized Additive Models

- More general again! Can do similar things to GLM.
- Fit a model using smoothing techniques, so they follow the data very closely.
- Non-Linear
- Problem: you can fit a great model to the data, but is it meaningful.

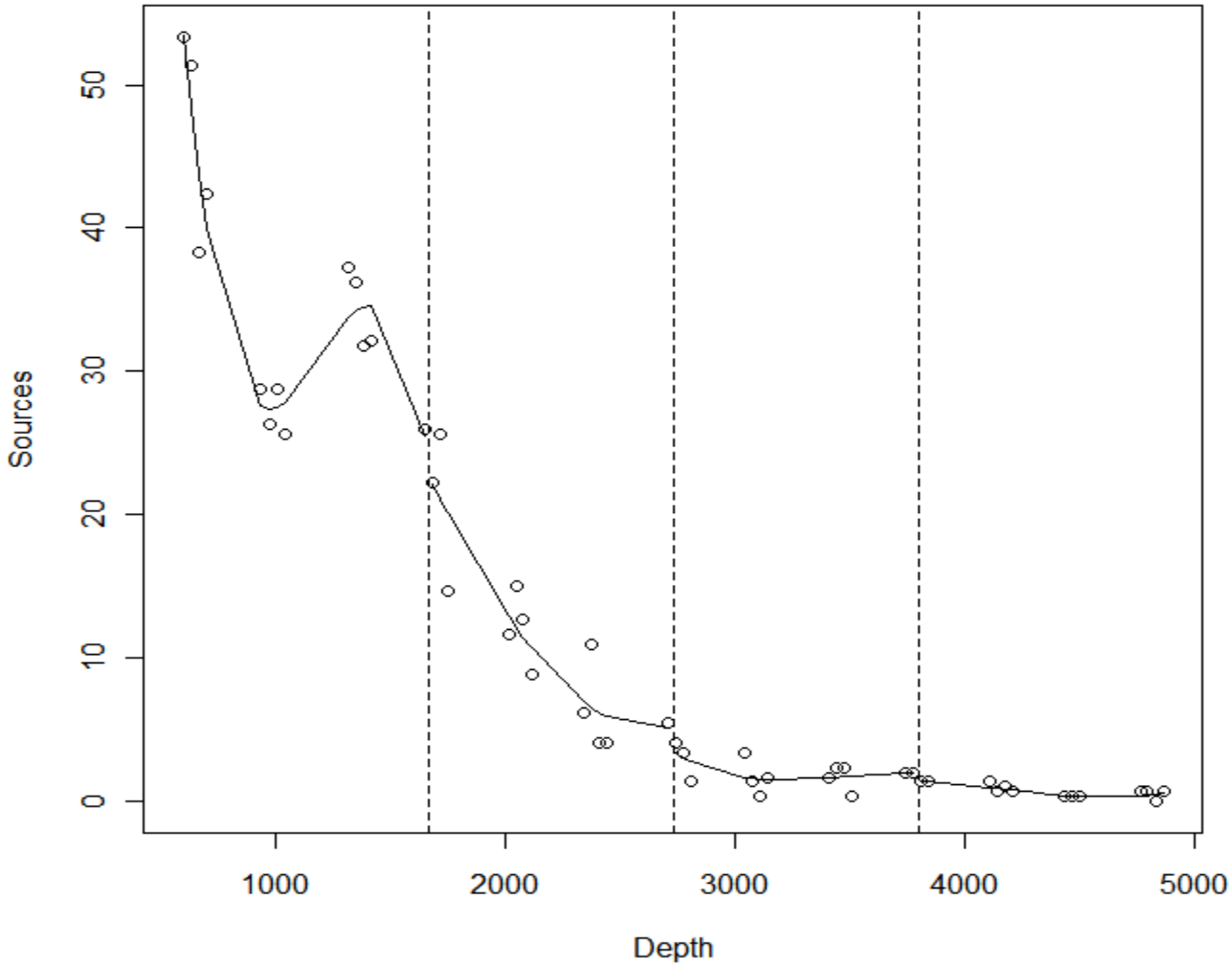
GAM cont.

- GAM has 3 parts
  - Data follows some dist e.g mass follows Poisson, mean = variance.
  - Link between mean of y (mass) and predictor variable(s). E.g. Log for poisson
  - Deterministic part:

$$\log(\text{mean roadkill}) = \alpha + f(\text{Dist.Park})$$

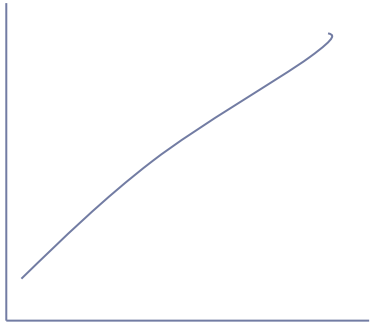
Smoother function

Example GAM smoother

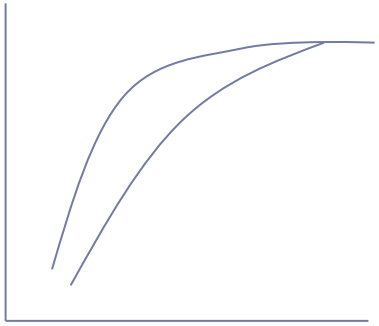


GAMM: Spatial autocorrelation shapes

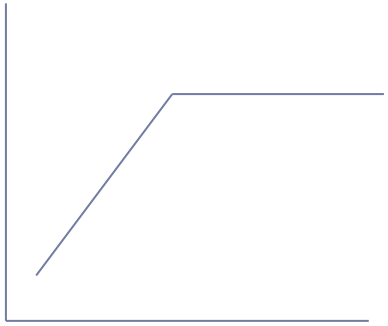
Ratio



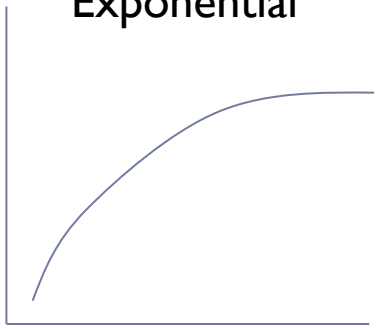
Spherical



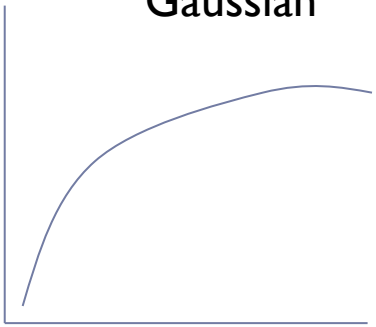
Linear



Exponential



Gaussian



Steps to choosing appropriate analysis

- What type of data is it? i.e. What distribution is most appropriate?
- Is the relationship linear or non-linear?
- Does the model have random variables, spatial or temporal confounding?

Further Reading